Announcements

1) Amanda's review session

2) Extra office hours

Wednesday 1:30–2:30

Thursday 10–12

Example 1: $f(x) = 9x^{2/3}(x+5)$

Find all critical numbers, intervals of increase/decrease, local maxima/minima, inflection points, intervals of concavity.

$f(x) = 9x^{2/3}(x+5)$

To find critical numbers,

find where f' is zero

or does not exist.

$$f'(x) = 9x^{2/3} + (x+5)6x^{-1/3}$$

$$= 3x^{-1/3}\left(3x + (x+5)\cdot 2\right)$$

$$= 3x^{-1/3}(5x+10)$$

$$= \frac{15x+30}{x^{1/3}}$$

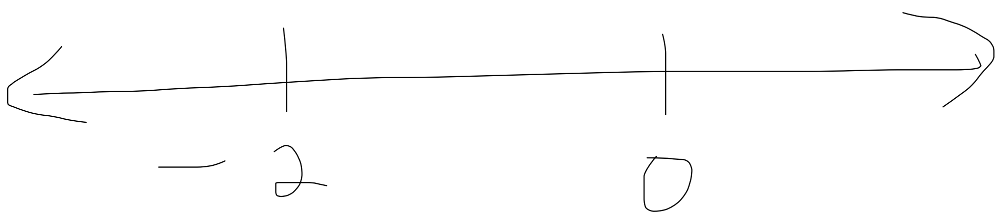
$f'(x) = 0$ when $x = -2$

f' undefined when $x = 0$

Critical Numbers: $x = 0, x = -2$

Intervals of Increase / Decrease

Plot all critical numbers



Intervals $(-\infty,-2)$, $(-2,0)$
$(0,\infty)$ plug in points
to $\boxed{f'}$

$(-\infty,-2)$ $x=-3$

$$f'(-3)=\frac{15(-3)+30}{(-3)^{1/3}}$$

$$=\frac{-15}{-3^{1/3}}>0$$

f is increasing

$(-2, 0)$ $\quad x = -1$

$$f'(-1) = \frac{-15+30}{(-1)^{1/3}} = \frac{15}{-1} < 0$$

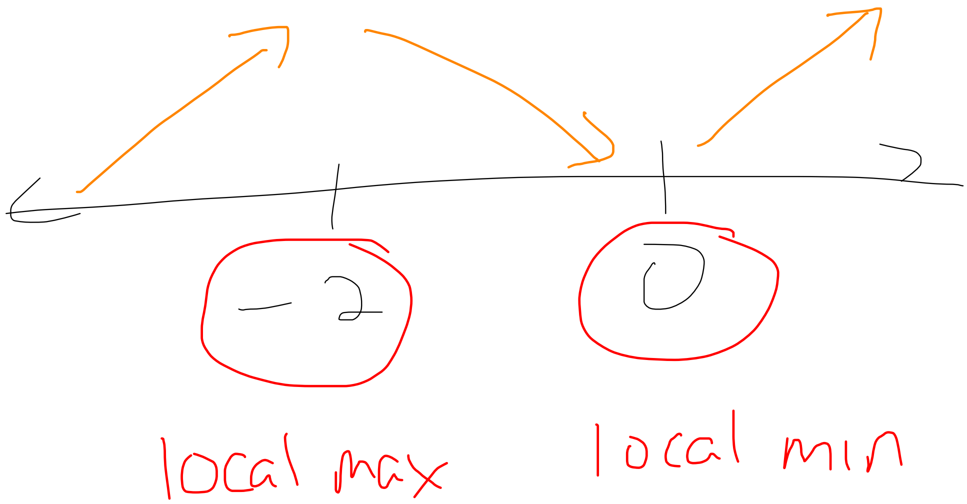
f is decreasing

$(0, \infty)$ $\quad x = 1$

$$f'(1) = \frac{15+30}{1} = 45 > 0$$

f is increasing

Intervals



Note: you only have local max/min when the points are in the domain of f.

Concavity / Inflection Points

Second derivative

$$f'(x) = \frac{15x+30}{x^{1/3}} = (15x+30)x^{-1/3}$$

$$f''(x) = (15x+30)(-1/3)x^{-4/3}$$
$$+ x^{-1/3} \cdot 15$$
$$= x^{-4/3}(-5x-10+15x)$$
$$= x^{-4/3}(10x-10)$$

Aside

$$(6x+3)x^{-4/3} + 2x^{-1/3}$$

$$= (6x+3)x^{-4/3} + 2x^{-4/3} \cdot x^{3/3}$$

$$= (6x+3)x^{-4/3} + 2x^{-4/3} \cdot x$$

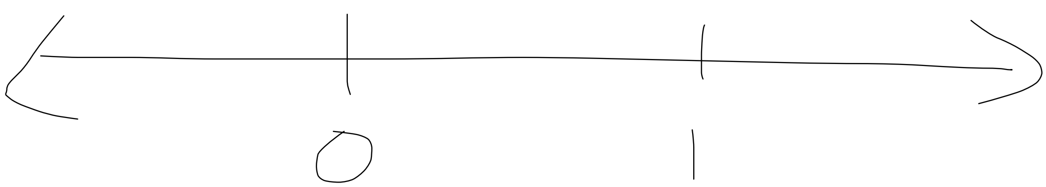
$$= x^{-4/3}(6x+3+2x)$$

$$f''(x) = \frac{10x - 10}{x^{4/3}}$$

$f''(x) = 0$ when $x = 1$

f'' undefined when $x = 0$

Plot these numbers



Intervals

$(-\infty, 0)$ $x = -1$

$$f''(-1) = \frac{-10-10}{1} < 0$$

f is concave down

$(0, 1)$ $x = \frac{1}{2}$

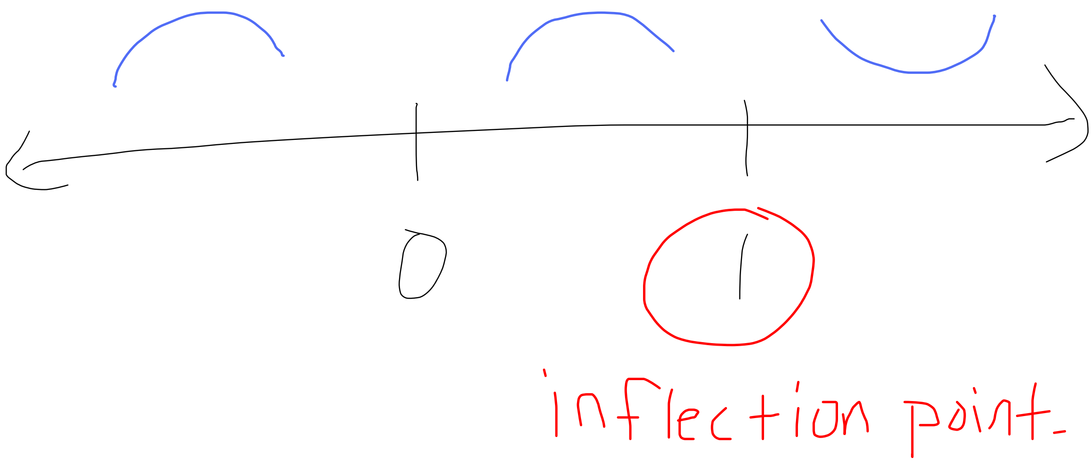
$$f''\left(\frac{1}{2}\right) = \frac{5-10}{\left(\frac{1}{2}\right)^{4/3}} < 0$$

f is concave down

$(1, \infty)$ $\quad x = 2$

$$f''(2) = \frac{20 - 10}{2^{4/3}} > 0$$

f is concave up



Example 2:

$$f(x) = \frac{3x}{\sqrt{4x^2+1}}$$